



The Montecarlo Method In The Teaching Of Financial Mathematics

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Abstract

The following work tries to show how through two mathematical models, one deterministic based on financial mathematics and the other probabilistic or stochastic based on the Monte Carlo method, the net present value of an investment project can be estimated.

Monte Carlo simulation is a statistical method used to solve complex mathematical problems through the generation of random variables (Eppen 2000).

The key to this method is to understand the term 'simulation'. Carrying out a simulation consists of repeating or duplicating the characteristics and behaviors of a real system. Thus, the main objective of the Monte Carlo simulation is to try to imitate the behavior of real variables in order, as far as possible, to analyze or predict how they will evolve.

Through simulation you can solve from very simple problems to very complex problems. Some problems can be solved with pen and paper. However, most require the use of computer programs such as Excel, Matlab, etc. Without these programs, solving certain problems would take a lot of time. (Wayne 2002).

Keywords: Montecarlo method, random numbers, financial mathematics

1. Introduction

The present work wants to estimate the money that could be obtained, in five years, to create a cookie company, therefore it begins to investigate various methods, which will give me an approximation of values, based on weekly savings.

when doing the investigation of different methods of financial savings, the variation of income, bank interest rates, among other variables, were taken into account, so that in addition to using a deterministic model,

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a probabilistic model should be applied to take into account the fluctuation of the variables and thus be able to estimate the nature of the different factors that can affect them.

Therefore, to put into practice the different aspects mentioned above, it is intended that, based on a complete theoretical framework, interest rates and payments are simulated, based on the Monte Carlo method and with series of uniform and variable payments (to uniform payments they are called annuities and gradient variables) find the future value of a given amount. Finally, it is expected to answer the question: How can the final value of an investment be estimated using the Monte Carlo simulation and payment series, both with annuities and with gradients?

Taking into account the above, for the following project a quantitative methodology will be used, based on the elaboration of an adequate theoretical framework. Subsequently, different interest rates in the market will be searched, analyzed and evaluated and financial evaluations will also be carried out in foreign currencies, with the aim of comparing the yields of one and the other. Finally, the project will be executed taking into account the weekly savings. It is important to emphasize that two mathematical models will be combined, one deterministic (series of uniform and variable payments) and the other probabilistic (Monte Carlo simulation, through random numbers).

2. Application Of The Model And Results

Next, the different ways to save and earn money will be evaluated, through savings accounts, CDT (Term Deposit Certificate) and currency exchange.

Savings account:

Table 1. Details of the best savings accounts with no handling fee. Font:(Rankia, 2019)

| Product | Interest Rate (%EA) | Cost X Month | Minimum Opening Amount |
|---|---------------------|--------------|------------------------|
| Saving My Account / Bancocompartir | 4.00% | \$0.00 | \$20,000.00 |
| Friend Savings Account / Caja Social Bank | 3.50% | \$6,960.00 | \$20,000.00 |
| Pro Profitable Account / Pro Credit Bank | 3.00% | \$0.00 | \$500,000.00 |
| Count Your Goal / World Woman | 3.25% | \$0.00 | \$0.00 |
| Programmed Savings Account / Banco Itaú | 2.75% | \$0.00 | \$0.00 |

Best Savings Accounts by Management Fee

Best CDTs for 2020: 90 days

| Entity | Products | handling fee | withdrawal cost | Minimum interest rate | Maximum interest rate |
|----------------------|-----------------------|-------------------------------|---------------------------------------|---|-----------------------|
| WWB Bank | basic savings account | no handling fee | no transaction fee | 1.00% EA — 3.20 EA starting at \$100,001 | 4.5% AE |
| Davienda Bank | Profitable Account | No Fee for the first 6 months | 6 free withdrawals \$1,350 additional | 0.10% AE | 4.45% AE |
| BankBBVA | AFC savings account | no handling fee | 0 | 0.12% AE | 4.5% AE |
| Bancamia | save me | no handling fee | \$1,810 | 0.25% AE | 3.00% A.E. |
| Bank Bogota | Savings income | \$10,500COP | \$1,500 | 0.10%EA | 1.80% AE |

Table 2. Data of the best savings accounts with handling fee. Font:(Rankia, 2019)**Annual effective rates with a cut-off date of 2020-01-15****Table1.** Data of the best CDT's at 90 days. Font:(Brosetta, 2020)

| Entity | Minimum amount | to 90 days. |
|---------------------------|----------------|-------------|
| Itau, Banco Corpbanca | \$1,000,000 | 4.85% |
| Bancocompartir SA | \$50,000 | 4.90% |
| oicolombia | \$100,000 | 4.95% |
| CF CA Financial Credit | \$250,000 | 5.10% |
| Bank W SA | \$1,000,000 | 5.20% |

Best CDTs for 2020: 180 days

Annual effective rates cut to 2020 - 01- 15

Table2. *Data of the best CDT's at 180 days. Font:(Brosetta, 2020)*

| Entity | Minimum amount | to 180 days |
|----------------------|----------------|-------------|
| Banco Pichincha S.A. | \$500,000 | 5.35% |
| oicolombia | \$100,000 | 5.50% |
| financial cotrafa | \$1,000,000 | 5.60% |
| BBVA Colombia | \$1,000,000 | 4.15% |
| Falabella Bank | \$500,000 | 4.30% |

Best CDTs for 2020: 360 days

Effective annual rates cut to 2020-01-15

Table3. *Data of the best CDT's at 180 days. Font:(Brosetta, 2020)*

| Entity | Minimum amount | to 360 days |
|-----------------------------|----------------|-------------|
| JFK Financial Cooperative | \$878,000 | 6.05% |
| Trust Financial Cooperative | \$439,000 | 6.10% |
| oicolombia | \$100,000 | 6.25% |
| financial cotrafa | \$1,000,000 | 6.30% |
| CA Financial Credit CF | \$250,000 | 6.35% |

Now, to take into account the exchange of currencies to euros and dollars, the values of the last day of the months from January 2019 to August 2020 were evaluated, the percentage was obtained by which the value of the currency decreased or increased with respect to the previous month, through the ratio of the value of the month, over the previous one, and finally it was multiplied by 100. An example of the above, with respect to what percentage the dollar increased from January 2019 to February of the same year:

$$\frac{3504}{3554,5} - 1 \times 100 = 1,42$$

In the end, the results were averaged, giving us an annual interest rate of 1.29% for the euro and 1.02% for the dollar. The calculations were made using Excel and the values for each month were obtained from the following pages:

Dollars:(Bank of the Republic, 2020)

Euros:(Investing.com, 2020)

Table4. Value of the euro and dollar, since January 2019, with its rate of increase. Source: the author

| Months | Price (euro) | rate of increase | Months | Price (dollar) | rate of increase |
|-----------|--------------|------------------|-----------|----------------|------------------|
| Jan-19 | 3,554.50 | | Jan-19 | \$3,163.46 | |
| February | 3,504.00 | -1.42% | February | \$3,072.01 | -2.89% |
| March | 3,576.50 | 2.07% | March | \$3,174.79 | 3.35% |
| April | 3,626.50 | 1.40% | April | \$3,247.72 | 2.30% |
| May | 3,780.00 | 4.23% | May | \$3,357.82 | 3.39% |
| June | 3,653.00 | -3.36% | June | \$3,205.67 | -4.53% |
| July | 3,634.00 | -0.52% | July | \$3,296.85 | 2.84% |
| August | 3,784.00 | 4.13% | August | \$3,427.29 | 3.96% |
| September | 3,793.00 | 0.24% | September | \$3,462.01 | 1.01% |
| October | 3,769.00 | -0.63% | October | \$3,389.94 | -2.08% |
| november | 3,875.50 | 2.83% | november | \$3,522.48 | 3.91% |
| december | 3,686.00 | -4.89% | december | \$3,277.14 | -6.96% |
| Jan-20 | 3,794.00 | 2.93% | Jan-20 | \$3,411.45 | 4.10% |
| February | 3,889.00 | 2.50% | February | \$3,539.86 | 3.76% |
| March | 4,479.00 | 15.17% | March | \$4,064.81 | 14.83% |
| April | 4,339.00 | -3.13% | April | \$3,983.29 | -2.01% |
| May | 4,140.00 | -4.59% | May | \$3,718.82 | -6.64% |
| June | 4,218.50 | 1.90% | June | \$3,758.91 | 1.08% |
| July | 4,397.50 | 4.24% | July | \$3,739.49 | -0.52% |
| August | 4,459.50 | 1.41% | August | \$3,760.38 | 0.56% |
| Average | | 1.29% | Average | | 1.02% |

Equivalence of rates.

Below is the conversion of the rates with the highest value of those previously presented, to a single period using the rate equivalence formula. This to compare its value in the same period of time. To obtain this result, the offered rates were equalized to a monthly period, since this is the period of time in which the currency rates are found. To do this, using the following equation, in which the variable i_2 , being the value of the presented rate: i_1

$$(1 + i_1)^{m_1} = (1 + i_2)^{m_2}$$

i_2 , being the value of the presented rate: i_1

$$(1 + i_1)^{m_1} = (1 + i_2)^{m_2}$$

$$(1 + i_1)^{\left(\frac{1}{12}\right)^{-1}} = i_2$$

Savings accounts:

In savings accounts with management fees, those that charged some type of value such as management fees, etc., were discarded. Staying as follows:

Table5. Equivalence of rates to one month, of savings accounts. Source: The author

| Entity | <u>Itau,</u> Banco Corpbanca | SA <u>Bancocompartir</u> | <u>oicolumbia</u> | CA <u>Financial</u> Credit CF | Bank W SA |
|--------------------------------|------------------------------------|-----------------------------|-------------------|-------------------------------------|--------------|
| to 90 days. | 4.85% | 4.90% | 4.95% | 5.10% | 5.20% |
| monthly equivalent rate | 0.40% | 0.40% | 0.40% | 0.42% | 0.42% |

Table6. Equivalence of rates to one month, of savings accounts with management fee. Source: The author

| Product | <u>Saving</u> My Account / <u>Bancocompartir</u> | Friend Savings Account / Caja Social Bank | Pro Profitable Account / Pro Credit Bank | Count Your Goal / World Woman | Programmed Savings Account / Banco Itaú |
|--------------------------------|--|--|---|--|---|
| Interest Rate (%EA) | 4.00% | 3.50% | 3.00% | 3.25% | 2.75% |
| monthly equivalent rate | 0.33% | 0.29% | 0.25% | 0.27% | 0.23% |

CDT's:

Table7. Equivalence of rates to one month of the CDT's to 90 days. Source: The autor

| Entity | WWB Bank | BBVA Bank |
|--|----------|--------------|
| Maximum Interest Rate (Annual Cash) | 4.50% | 4.50% |
| monthly equivalent rate | 0.37% | 0.37% |

Table8. Equivalence of rates to one month of the CDT's to 180 days. Source: The author.

| Entity | JFK Financial Cooperative | Trust Financial Cooperative | <u>oicolumbia</u> | <u>financial</u> <u>cotrafa</u> | CA <u>Financial</u> Credit CF |
|--------------------------------|---------------------------------|-----------------------------------|-------------------|------------------------------------|--|
| to 360 days | 6.05% | 6.10% | 6.25% | 6.30% | 6.35% |
| monthly equivalent rate | 0.49% | 0.49% | 0.51% | 0.51% | 0.51% |

Table 11. Equivalence of rates to one month of the CDT's to 180 days. Source: The author.

| Entity | Banco Pichincha S.A. | <u>oicolumbia</u> | BBVA Colombia | Falabella Bank | <u>financial</u> <u>cotrafa</u> |
|--------------------------------|-------------------------|-------------------|------------------|-------------------|------------------------------------|
| to 180 days | 5.35% | 5.50% | 4.15% | 4.30% | 5.60% |
| monthly equivalent rate | 0.44% | 0.45% | 0.34% | 0.35% | 0.46% |

Final value or desired amount

1. A weekly saving that varies between \$10,000 and \$20,000 was evaluated, assuming that each month this value was changed to Euros, taking into account the average increase in the euro in the last 20 months, as was done in table 6.
 - 1.1. The modification that will be made in this section is that the euro exchange rate will also vary between -5% and 5%, throughout the 60 months.
2. Between \$10,000 and \$20,000 per week will be saved, which at the end of each month will be invested in the savings account of the WWB, which is one of the banks that pays the highest rate

(0.3675%), without a management fee, and although BBVA does not offer a similar interest rate, WWB does not offer a higher minimum. Since the payments are uniform and of equal value, they are an annuity, so the final value formula will be used, of the same:

$$VF = R \frac{(1 + i)^n - 1}{i}$$

Where: final value $VF =$

R =value of each payment

i =interest rate

n =number of periods in the year.

3. In the same way, it will be deposited every six months to the CDT of CA Credifinanciera CF, for being the one that provides the most monthly interest (0.5144%).

4. Finally, the values of the CDT savings account will be evaluated in an arithmetic and a geometric gradient.

To develop the first situation, the Monte Carlo model was applied by generating random numbers in Excel in an interval of 1 to 10. Subsequently, that value was multiplied by 1,000, and finally by 4, to have the value saved in the month. An example of the above, where 5 is the value obtained by the Monte Carlo simulation, and \$200,000 is the value saved in one month.

$$5 \times \$1.000 \times 4 = \$20.000$$

For euros, we multiplied the current value of the euro by a constant value of 0.1, expecting that to increase each month, and divided by the value saved in pesos. This was done for 60 months, and the sum of the total value in euros was equivalent to €727.793687, and when multiplied by the value of the euro in COP estimated in 60 months, the final value was \$49,489,970.72. The Excel table where the random values were applied is found in Annex 1.

Now, to vary the percentage increase of the monthly euro, the Montecarlo model was applied in the same way, using Excel, random numbers between -0.5 and 5 were generated, then multiplied by, resulting in the respective percentage of increase or decrease of the monthly euro. The first value was taken as a reference for the month of August, equivalent to \$4,459.50, and for the following months the value of the previous month was multiplied by the sum of 1 and the rate. Next, the example with the value of month 2 will be shown. 10^{-2} ,

$$\$ 4.459,50 \times (1 + 5\%) = \$ 4.682,48$$

(5% is the random value of the rate increase).

Already having the corresponding values of the variation of the euro in each month, the same procedure of the first situation was carried out, to find the final value after 5 years, the sum of the 60 months was made, after applying the corresponding variation of the euro, resulting in €900.32 and when converted to COP it was \$72,025,408.18. The Excel table where the random values were applied is found in Annex 2.

For the second and third situations, the formula described above was applied for savings accounts and CDTs, respectively, as follows:

$$VF \text{ Cuenta de ahorros} = 60000 \frac{(1 + 0,3675)^{60} - 1}{0,3675} = \$ 4.019.528,77,$$

$$VF \text{ CDT} = 60000 \frac{(1 + 0,5144)^{60} - 1}{0,3675} = \$ 4.204.845,12$$

R , was chosen as 15,000, since it is an intermediate value between the proposed interval.

As evidenced in the formula, the total value, for constant savings in an account, for the five years in total \$4,019,528.77, modifying by 60 months. For the CDT, the savings were \$4,204,845.12. n

For the last situation, Gradient Series were used. A value was established to increase annually, which was \$520,000, increasing \$10,000 each week. For the geometric gradient, given its definition, it was established to increase the value by 40% per year. So at Applying the arithmetic gradient formulas for final value after a period of 5 years, resulted in \$12,477,656.29 for the savings account and \$17,556,712.07 for the CDT. The corresponding formulas are the following:

$$734.732,76 \frac{(1 + 0,3675)^5 - 1}{0,3675} + \frac{520.000}{0,3675} \left[\frac{(1 + 0,3675)^5 - 1}{0,3675} - 5 \right] = \$ 12.477.656,29$$

$$740.723,60 \frac{(1 + 0,5144)^5 - 1}{0,5144} + \frac{520.000}{0,5144} \left[\frac{(1 + 0,5144)^5 - 1}{0,5144} - 5 \right] = \$ 17.556.712,07$$

Now, when finding the final value of five years of the geometric gradient, the formula was applied, resulting in \$13,472,547.20 for the savings account and \$16,750,945.58 for the CDT. The formulas used were the following, respectively:

$$\frac{\$ 734.732,76 [(1 + 0,4)^5 - (1 + 0,3675)^5]}{0,4 - 0,3675} = \$ 13.472.547,20$$

$$\frac{\$ 740.723,60 [(1 + 0,4)^5 - (1 + 0,5144)^5]}{0,4 - 0,5144} = \$ 16.750.945,58$$

Taking into account the results presented, we can conclude that after 5 years the savings strategy which will give me the most money at the end of time will be the exchange to foreign currencies, or to Euros every month, with a final value of \$72,025,408.18

Costs and income of the company.

As mentioned at the beginning of the document, with that savings a cookie company will be started, so we will estimate the possible expenses that will be needed, in order to maximize the results. Therefore, the values of the expenses will be presented below.

Ingredients to make 16 cookies (1 batch):

- 200 grams butter
- 120 grams sugar
- 280 grams of wheat flour
- 1 teaspoon vanilla essence

Next, it will be specified:

1. The wholesale costs in the market of the respective products and their quantities.
 2. The number of product units to make more than 400 batches and their price.
 3. The final number of so many that can be made.
- 200g butter
 1. ArtePan Butter 15000 grams \$73.124
 2. 6 units are needed, which would cost \$438,750
 3. $90.000g \div 200g = 450$ tandas
 - sugar 120g
 1. Common Sugar Lump X 50000 grams \$119,990

2. One unit of product is enough
 3. $50.000g \div 120g \approx 416$ tandas
- Wheat flour 280g
 1. Tres Castillos Wheat Flour Bulk X 50000 grams \$96.950
 2. 3 units are needed, which would cost \$290,850
 3. $150.000g \div 280g \approx 535$ tandas
 - 1 teaspoon (5C.C) vanilla essence
 1. Egg World Black Vanilla Essence X 500 CC \$5,400
 2. 4 units are needed, which would cost \$21,600
 3. $2.000g \div 5 = 400$ tandas

Taking the above into account, we can say that the total cost to carry out approximately 400 batches is \$871,190, so each batch would have a value of \$2,177,975, that is, approximately \$2,200.

Now, packages of 8 cookies are to be sold at a price of \$1,100, so each batch will produce 800 packages. Each pack will be sold to the public for \$4,000, for a profit of \$2,900.

Bearing in mind that the cookies and the business will begin at home, where sometimes you will have to buy ingredients at retail, or make an additional payment to an occasional helper, it was estimated that the earnings of \$2,900 would be they were going to decrease \$200 above a production of 800 packages.

Therefore, a formula will be proposed where the number of cookies necessary is found so that there are no losses, taking into account the additional costs.

$$U = 800 \times 2900 + x \times (2900 - 200x)$$

x = The number of cookies produced above 800 packets and is the profit obtained, so we need to find the value of x at which maximum profit is obtained.

Differentiating U with respect to x , we obtain:

$$U = 800 \times 2900 + x \times (2900 - 200x)$$

$$\frac{du}{dx} = 2900 - 400x$$

$$-2900 = -400x$$

$$\frac{-2900}{-400} = x$$

$$x = 7,25$$

Given the results obtained, to obtain a maximum profit, at least 808 packages must be produced and in this way, be able to obtain sufficient profits from the company, and have an income in my future.

3. Conclusions and Discussion

Throughout this project, it was possible to estimate the best way to make savings in 5 years, by comparing interest rates of different savings methods in the country. As a result, it was shown that, by exchanging foreign currency into Euros, taking into account its performance in the last 20 months, it is the one that generated the most income. It was possible to estimate an income of \$72,025,408.18, at the end of 5 years, taking into account variations in the rise or fall of the dollar, and also the monthly income from savings. Therefore, the objective of completing the objective was fulfilled by using the deterministic and probabilistic models, so that, through the comparison of saving methods, the best option was found. It should also be noted that this amount is enough to start a cookie company, with a few minimum ingredients, and thus achieve maximum use of resources to avoid having any loss. It is expected to apply the results of this work, to start saving money, taking into account the results presented.

It should be noted that this project contributed to my personal life, because, through tests, equations and simulations, it was possible to arrive at an estimate of savings, of my money and how to invest it in the best way. In this way, I can be confident with the saving method. On the other hand, I can apply the equations of equivalence of rates, so that in the future, I will know the best option to save, also to know the final value of a savings or debt, through the formulas presented, that best fit the situation. This will help me to be more aware of my expenses, to control them, and thus have a better management of my finances. Likewise, I am clear about how to start a company, how to plan expenses and income, and thus maximize resources, to avoid any type of loss.

Finally, it can be affirmed that people do not save knowing that they can earn money through different methods, mainly due to the lack of knowledge that exists about the financial system, about the management of interest rates, among others. On the other hand, there is the fact that in Colombia the banks offer too low a rate when making savings, compared to those charged on a loan; for this reason it is better to save money, and take advantage of it by exchanging currencies.

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