Available online at www.jlls.org
JOURNAL OF LANGUAGE AND LINGUISTIC STUDIES
ISSN: 1305-578X
Journal of Language and Linguistic Studies, 18(2),1133-1142; 2022

Development of Variational Thinking Based on Non-Routine Problem-Solving in Elementary School Students

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APA Citation:

Suarez, C.A.H., Castro, W.R.A., Suárez, A.A.G. . (2020). Development of Variational Thinking Based on Non-Routine Problem-Solving in Elementary School Students. *Journal of Language and Linguistic Studies, 18*(2), 1133-1142.

Submission Date: 20/10/2021
Acceptance Date: 25/01/2022

ABSTRACT

This article aims to determine the effect of a pedagogical intervention aimed at developing variational thinking in elementary school students. The research approach is quantitative at a descriptive level, where an analysis of the teacher's pedagogical practice was carried out through a pretest-posttest design. According to the diagnostic test results, a transversal project supported by a pedagogical intervention was prepared to subsequently apply the test to determine the impact of the intervention. The results show that some children could reach the level of abstraction and algebraic modeling of problems through the formulation and resolution of equations and the use of tables and graphs, which made it possible to prove that variational thinking can be developed from the first grades of elementary school. It is concluded that relating the student's environment and using concrete material allows them to understand the applicability of concepts related to variational thinking.

Keywords: elementary education; variational thinking; problem-solving.

INTRODUCTION

Variational thinking is one of the types of mathematical thinking related to the ability to identify, perceive, model and interpret situations and processes to identify changes, variables and patterns in daily life (Mineducación, 1998). Likewise, this is linked to metric, spatial, numerical and random thinking and with thought processes of other sciences (Mineducación, 2006) and together with these thoughts, it is essential to generate the ability to reason, solve and pose problems, communicate and model, integrally promoting the teaching of mathematics without neglecting the role of each of the thoughts that integrate it.

For such reasons, its development can be initiated in the first grades of primary education within the mathematics curriculum (Mineducación, 1998) since the mental processes involved are basic in children's mathematical development. The pedagogical actions generated for its development should improve students' mathematical skills from everyday situations for students that allow them to develop the ability to solve problems in contexts and demonstrate the useful, practical and meaningful sense to then understand more advanced thinking processes. It is from this perspective that this study was approached.

In particular, Mineducación (1998) states that the development of variational thinking is one of the achievements to be reached in basic education in which two characteristic and relevant elements, such as change and variation, are interrelated. Likewise, the development of variational thinking is proposed from the first years of schooling to overcome the teaching of fragmented and compartmentalized mathematical contents to be located in the domain of a conceptual field that involves inter-structured concepts and linked procedures (Mineducación, 1998). In this way, students would have the ability to be mathematically competent by enhancing their ability to analyze, organize and model everyday situations so that they identify and understand the role played by mathematics and science in the world (Gómez, 2020).

Therefore, the importance of each of the thoughts and especially variational thinking in the first years of schooling, is highlighted, so the study was oriented to identify the difficulties in its teaching, to adapt to the needs of students a strategy mediated with their daily context in order to develop it. Considering this, the development of variational thinking has become a relevant topic for researchers and educators; this thinking allows students to improve their understanding and interpretation of phenomena related to their environment. As Hecklein et al. (2011) pointed out, enhancing or developing variational thinking implies preparing students to solve problems and deal with the information they receive from the environment so that they can recognize the strategies for their solution and favor a better understanding and interpretation of reality. In this sense, the processes of variation and change constitute an aspect of great richness in the school context.

Furthermore, as pointed out by Mineducación (2006), the development of variational thinking, given its characteristics, is slow and complex but indispensable for characterizing aspects of variation such as what changes and what remains constant, the variables involved, the field of variation of each variable and the possible relationships between these variables.

Hence, as mentioned by León-Salinas (2016), it is very common for students to ask questions about changing phenomena that they see around them; the level of rainfall, for example, the temperature or the salary of their parents are some of these phenomena that can become enhancers of activities that study these changes. Therefore, variational thinking seeks to favor the learning process of mathematics by inducing the student to reason, which implies the development of logical thinking that incorporates intuitive and inductive reasoning to find solutions to the proposed problem situations. Therefore, this study opted for integrating a transversal project that would allow students to link mathematical knowledge and understand its applicability in situations generated in the context, which seeks to facilitate cognitive processes and contribute to their comprehensive education.

This is achieved through strategies that allow a transformation of traditional methodologies through a classroom project, which is an alternative strategy to the traditional methodology, because it allows for generating learning environments, which arise from the interests and needs of students and encourages group work, active participation, reflection and creativity; it also allows the contextualization of a mathematical concept in a meaningful way, finding its usefulness in everyday life situations (Cubillos & León, 2016).

In this sense, in order to make the concepts of variation and change fundamental in the development of variational thinking, it was necessary to include the use of concrete material, leading students to carry out an implicit process starting with the identification of patterns, followed by representation, abstraction and finally generalization. For this reason, a series of pedagogical interventions were proposed, where concrete materials present in the context are manipulated, in addition, a transversal project is integrated, which is the growth of a plant and based on this to achieve that the phases required for the development of variational thinking are carried out, reinforcing aspects that influence the progress of other thoughts and thus contextualizing the learning that allow the development of competencies necessary for children to interact in their environment dynamically, taking what was learned at school to their real life.

It is worth highlighting that one of the important factors in this process was to emphasize the use of representations, a feature of great importance in the study of variational thinking, representation systems, which improve the understanding of mathematical events (Prada-Núñez et al., 2016); this allows students to understand, analyze and interpret information more easily as representations play a very important role since representations are something inherent to them, which significantly improve understanding in students (Prada et al., 2017).

On the other hand, despite the progress of technology and innovative pedagogical strategies used in the classroom, mathematics continues to be a difficult subject in the academic process. It is the one that generates more apathy and feelings of frustration in students, which Fernández-Cézar et al. (2020) call mathaphobia. Therefore, it is necessary to propose new teaching strategies in accordance with students' needs and consider the situation of constant changes in which we live. Students can build mathematical knowledge with the support of ICT, where good results are obtained and are already evident through the variational aspects, the use of graphs, design experiments and the use of technology as a tool allows them to manipulate graphs more easily (Briceño & Buendía, 2016).

Likewise, Ordóñez-Ortega et al. (2019) bet on the use of mediated with algebraic tiles and virtual manipulators to motivate students in the use of concrete material, where an improvement in the development of variational thinking was evidenced, suggesting that students will have better performance. Additionally, this allows changing the perspective of mathematics, thus allowing the enhancement of mathematical competencies and thoughts, leaving aside the algorithmic way with which the contents and processes are assumed in the classroom that generate apathy in students, as proven by Hidalgo et al. (2013), who confirms that at the beginning of schooling students start with competencies and interest to address problem-solving much higher than those presented as time goes by, since this gradually decreases due to the demotivation that generates to the student the how and what he learned because he does not find meaning or usefulness within his context.

In the same way, teachers must have the capabilities to adapt and adopt different tools and strategies that allow the construction of knowledge in the classroom. Likewise, the conceptual domain of the teacher is an influential factor in student learning, as identified by Caballero & Cantoral (2013a, 2013b). They determine that one of the greatest difficulties of teachers lies in the limited knowledge regarding the domain of variational thinking, identifying the conceptual and didactic gap that teachers have to proceed in teaching this thinking.

Finally, it is worth highlighting the importance and influence of the adequate and conscious design by the teacher of the pedagogical interventions in the teaching-learning process of mathematics (Gamboa, 2016; Castañeda et al., 2020); an adequate learning process is achieved when starting from the diagnosis of pre-knowledge that guides the subsequent work, additionally performing a continuous evaluation allows improvements in the learning processes (Gamboa, 2019). For example, Martínez-López & Gualdrón-Pinto (2018), through didactic sequences mediated by ICT organized in three stages (diagnosis, interventions and final diagnosis), managed to evidence that the change of methodology generated better learning environments and increased student participation, which guided by the didactic engineering approach presents a series of phases to address the didactic work where it takes into account: (a) Preliminary analysis; (b) Conception and a priori analysis of didactic situations; (c) Experimentation; (d) A posteriori analysis and evaluation, whose purpose is to optimize and increase the scope of favorable results in the teaching process (Campeón et al., 2018), as will be evidenced later in the results obtained in the present research.

METHOD

In this research, all the research instruments and techniques will be used based on a quantitative, descriptive and cross-sectional approach during the first semester of 2022.

A population comprised of all students enrolled for that school year in an educational institution of basic and secondary education in the department of Norte de Santander, Colombia, and a sample selection by the technique of non-probabilistic sampling by convenience, for a total of 15 students, where some parameters were established to select the students belonging to the sample.

Research design

The study focused on the development of variational thinking and the mastery of communication, problem-solving and reasoning processes that influence students' performance in mathematics through three stages: preliminary analysis, identification of difficulties and evaluation of the pedagogical impact.

Stage 1 Preliminary analysis.

It consisted of a background search on variational thinking to confront the institutional mathematics area plan with Mineducación's curricular guidelines, such as standards and basic learning rights (Mineducación, 1998, 2006, 2016).

Stage 2 Identification of difficulties.

The curricular guidelines (Mineducación, 1998) were taken as a basis. A test of knowledge was created consisting of 21 questions, created in response to specific learning in the last grade of elementary school, which transits to the following school year towards secondary education in terms of variational thinking, as well as the creation of 4 pedagogical interventions applied weekly to develop the processes of variational thinking through activities such as the study of patterns, representation systems, abstraction and generalization. In this planning, the context was integrated, concrete materials that the students could manipulate were implemented, and a transversal project on plant growth was implemented.

Stage 3 Evaluation of the pedagogical impact.

The knowledge test was applied again and inferential statistics were used to contrast the results at the two different times (before and after the pedagogical interventions) to determine if there were significant differences between the grades obtained by the students in the two measurements, implementing the hypothesis test of differences in means for paired samples.

RESULTS

Stage 1 Preliminary analysis. Comparison between what has been proposed by the Ministry of Education and what has been implemented in the Educational Institutions

The following is a parallel between what is proposed by Mineducación for the orientation of mathematics learning and how these processes are carried out in the educational institution under study, focused on the fifth grade of elementary school.

Table 1. The contrast between what was proposed by the Ministry of Education and what was implemented in the educational institution.

Ministry of National Education of Colombia	**Educational Institution**
At a general level on the teaching of mathematics	
Development of the five thoughts: numerical, metric, spatial, variational and random.	The educational institution has in its work plan the separation of the area of mathematics, the thoughts are not associated and there is no transversality between them.
Development of mathematical competencies	In practice, the teacher is guided by texts (books) for teaching and through this procedure, a rote learning is achieved; as a consequence, students do not acquire skills and in turn do not develop the minimum competencies required.
Work with mathematical processes: reasoning, communication, modeling, exercise and problem-solving.	Mathematical processes such as reasoning, communication and exercise are developed in the educational institution. However, there is no emphasis on modeling and problem-solving. In the case of problem-solving, it is carried out methodically and no strategy is applied to its solution.
Starting from meaningful and comprehensive learning situations of mathematics.	Virtual education was implemented in the institution with tools such as Classroom and WhatsApp, through which the teacher shares guides and workshops and the students send them resolved; before the health emergency, students came from a traditionalist learning with some knowledge constructions.
Design learning processes mediated by cultural and social scenarios.	The spaces available and the local context are not considered when designing the learning processes, despite having the space to do so.
To foster in students attitudes of appreciation, security and confidence towards mathematics.	There is no evidence of moments in a class, it does not begin with an exploration of learning to identify past experiences with the learning to be addressed, which can cause frustration and confusion in children.
Overcoming the stability and inertia of teaching practices	Learning tends to be static and rote, there is no practice innovation and obsolete books guide it.
Exploit the variety and effectiveness of material resources (physical or virtual supports) implementation of ICTs	The institution does not have the technological resources for all students, in addition to virtual education, most children do not have internet or technological means to connect, so they simply send guides via WhatsApp.
Evaluate in a formative manner	In the case of the educational institution, the evaluation process does not take into account the progress and achievements of the student in the process, but rather the evaluation is summative, which implies that learning is not adapted to the needs or shortcomings of the student.
In terms of variational thinking	
Expand the study of variation phenomena in particular when related to proportionality and the use of the properties of natural number systems and fractions to construct non-conventional procedures to solve equations.	The concept of variation or variable dependence is not established; according to what is evidenced in the students' notebooks, a sequence of steps for solving equations is proposed, but no applicability allows them to construct graphs or tables. As for patterns, both numerical and geometric sequences are handled.

Stage 2 Identification of difficulties. Analysis of pedagogical interventions.

Table II shows the strengths and weaknesses that were evident in developing the four pedagogical interventions. Also, the impact of the transversal tree growth project on the children.

Table 2. Analysis of the pedagogical interventions.

Pedagogical Intervention I	
Strengths	**Weaknesses**
Both geometric and numerical patterns were analyzed and interpreted. It was transversalized with the other mathematical thoughts, especially with the numerical and metrical.	There was not enough physical space to develop the planning and materials for modeling, especially the board. Schematization, table construction, among others.
Pedagogical Intervention II	
Strengths	Weaknesses
The concrete materials were eye-catching, easy to manipulate, and appealing to the students. The students well received the cross-cutting project.	For intervention II, cooperative work had been planned so that in groups, they could handle the concrete material, but it was impossible to work with a large group of students, so all the children worked with the same material, and less time was used than planned.
Pedagogical Intervention III	
Strengths	Weaknesses
Understanding of the concept of the equation using the balance was achieved. Some children posed equations regarding the growth of their plant.	Few specific materials were available for the group of children with whom we worked. The planning stipulated cooperative work, but due to the pandemic, it could not be carried out in this way because of the number of children attending.
Pedagogical Intervention IV	
Strengths	Weaknesses
The guide served as a basis for equations to be posed and for solving other proposed problems. The relationship of the interventions made with the cross-cutting project was found. Some students achieved the general mathematical process of problem solving.	Some students provided solutions to the problems presented for intervention IV, but they did not perform an equation approach nor provide solutions to the equations.

Stage 3 Evaluation of the pedagogical impact. Evaluation of the test in two stages.

Table III shows the test results at two points in time, one before the pedagogical interventions and the other after them, as well as the difference between these results. Again, the scores have been given considering the student's handling to arrive at the correct answer.

Table 3. Difference between pretest and posttest scores.

Students	Pretest	Posttest	Difference = posttest - pretest
E_1	11.0	15.0	4.0
E_2	6.0	8.0	2.0
E_3	10.5	16.0	5.5
E_4	7.0	12.0	5.0
E_5	2.5	8.5	6.0
E_6	10.0	14.0	4.0
E_7	13.5	17.5	4.0
E_8	5.0	10.5	5.5
E_9	7.5	14.5	7.0
E_10	11.5	13.5	2.0
E_11	13.5	17.5	4.0
E_12	8.0	10.5	2.5
E_13	11.0	15.0	4.0
E_14	5.0	10.5	5.5
E_15	7.0	12.0	5.0

Scoring.

- A score of one point is assigned if the answer is correct and the procedure performed is adequate.
- Half a point is assigned if the answer is reached but without the correct procedure or half procedure.
- No grade is assigned if the answer is incorrect.

The hypothesis test for means with paired samples is used to evaluate the impact of the pedagogical interventions on a sample of students in the fifth grade of elementary school of the educational institution under study. The comparison of the same test, applied at two different times, one before the pedagogical interventions (pretest) and the other afterward (posttest), was carried out. The value of $t = 11.72$ that has been calculated is greater than $t_\propto = 1.76$; therefore, the null hypothesis, where there is no difference between the results of the posttest and pretest, is rejected.

It can be said that, with this level of significance, there is sufficient statistical evidence to consider that the pedagogical interventions in the sample of students worked and allowed them to improve the development of variational thinking, as well as their academic performance in this thinking and mathematics in general, carrying out pattern discovery processes.

DISCUSSION

With the results obtained, it is possible to analyze the gap between what is proposed by the Colombian Ministry of National Education, the mathematics area plan of the Institution, and the teacher's pedagogical practice. Teachers have a frame of reference but also a teaching autonomy to define what learning should be prioritized according to the context; however, many of the conditions that should be taken into account for the teaching of mathematics and especially for the case of the development of variational thinking are not being met. The needs of the context are not taken into account, nor are they brought into play in the teaching and learning processes. The processes are based on memorization and obsolete texts and books. As for evaluation, it is not done in a productive way, where feedback is given on learning that has not been assimilated, which allows progress to be made, but with gaps that may later affect the processes of students in higher education grades. These results are similar to those proposed by (Hernández et al., 2021).

Currently, in most educational institutions, primary school teachers are guided textually by the book and no spaces are allowed during the development of classes for analysis, argumentation and discussion, even though most are based on a traditionalist model; this is supported by Jiménez & Gutiérrez (2017) where teachers do not involve the context of students in their processes and there is no implementation of concrete materials. Likewise, their method is based on memorization. As a result, the educational levels of Colombian educational institutions, especially at the basic level, have many gaps and incomplete and few novels and dynamic processes, which is why low results are obtained in standardized tests at the national (Velásquez-Luna et al., 2017) and international level (Demarchi-Sánchez, 2020).

To plan and execute a pedagogical intervention, the teacher must have technical and disciplinary knowledge and also guide the process well to reach the objectives expected to be achieved through these interventions. However, the main strengths of the pedagogical interventions implemented in this research and planned based on the results of the diagnostic test were the attraction they generated in the students since they manipulated many concrete materials, they were seen as different “classes,” where they found another way to learn what they knew in one way or another, In addition to seeing the applicability of the learning that was developed, results were also achieved in the students in a short time and with materials that were easy to obtain, confirming what Touriñán (2019) establishes as an intentional action that is developed in the educational task in order to carry out with, by and for the learner the ends and means that are justified based on the knowledge of education.

As for the transversal project, it was very striking for the students because it generated interest in determining the growth of the plant for the time that elapsed, which involved being aware of its care for its growth and correctly filling out the registration form that was given to them, to make comparisons and reach conclusions finally. Pedagogical interventions have a positive effect on mathematics education and mainly on the strengthening of variational thinking, as also proved by Ordóñez-Ortega et al. (2019), where the interventions obtained favorable results in terms of the assimilation of concepts of variation and dependence of variables and the change of attitude towards mathematical learning when not done by the traditional method was noticed. According to the above, the use of transversal contents, such as environmental protection, can be supported with mathematics, as also evidenced by Peralta & Peña (2017) and Pedreros & Peralta (2018).

Finally, the evaluation of the impact of the interventions showed that they had a positive effect, which strengthened variational thinking in the sample of fifth-grade students and mathematical processes, especially communication, reasoning and problem-solving. This will have positive effects on subsequent learning at the secondary and middle school levels, which reflects the importance of the development of algebra and variational thinking in early education, in addition to demonstrating that students can conceive concepts related to variation and change even in the first and second grades of elementary education as evidenced by Morales et al. (2018).

CONCLUSIONS

It was evidenced that, despite the curricular guidelines proposed by the Colombian Ministry of National Education for the teaching of mathematics, the teachers at the elementary school level of the educational institution under study exclude from the teaching process competencies necessary to develop the students’ learning or are not adequate, nor do they comply with the established conditions, especially in the case of variational thinking.

On the other hand, the implementation of the proposed pedagogical interventions and the cross-cutting project allowed to verify that the students show a positive attitude towards the implementation of concrete materials and the use of elements present in their context for learning, generating a better learning environment for the students where it was shown that assertive participation could be achieved when interest is encouraged. Moreover, the spaces and resources available are taken advantage of.

Statistically, it was possible to evaluate the effect of the proposed pedagogical interventions and the transversal project since the results of the knowledge test applied before and after were compared, allowing to corroborate that they worked and strengthened the development of variational thinking in the selected sample of students.

Finally, it can be concluded that the students were able to develop mathematical competencies focused mainly on variational thinking, which is why it is necessary to say that the teaching and learning processes should be related to their environment, through concrete materials, simple to manipulate and eye-catching in order to be more easily understood. This is expected to improve the results in the Saber tests and good development of variational thinking, useful at high school and middle school levels.

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